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Design and optimization for variable rate selective excitation using an analytic RF scaling function

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Abstract

At higher B_0 fields, specific absorption rate (SAR) deposition increases. Due to maximum SAR limitation, slice coverage decreases and/or scan time increases. Conventional selective RF pulses are played out in conjunction with a time independent field gradient. Variable rate selective excitation (VERSE) is a technique that modifies the original RF and gradient waveforms such that slice profile is unchanged. The drawback is that the slice profile for off-resonance spins is distorted.

A new VERSE algorithm based on modeling the scaled waveforms as a Fermi function is introduced. It ensures that system related constraints of maximum gradient amplitude and slew rate are not exceeded. The algorithm can be used to preserve the original RF pulse duration while minimizing SAR and peak b1 or to minimize the RF pulse duration. The design is general and can be applied to any symmetrical or asymmetrical RF waveform. The algorithm is demonstrated by using it to (a) minimize the SAR of a linear phase RF pulse, (b) minimize SAR of a hyperbolic secant RF pulse, and (c) minimize the duration of a linear phase RF pulse.

Images with a T1-FLAIR (T1 FLuid Attenuated Inversion Recovery) sequence using a conventional and VERSE adiabatic inversion RF pulse are presented. Comparison of images and scan parameters for different anatomies and coils shows increased scan coverage and decreased SAR with the VERSE inversion RF pulse, while image quality is preserved.

Keywords: SAR reduction; Analytic RF scaling function; Constant pulse width; Constant contrast

1. Introduction

RF power deposition per unit weight (Specific Absorption Rate or SAR) scales roughly as the square of the B_0 field. Therefore, high SAR can severely limit scan time and slice coverage in body imaging applications at higher field strengths.

Various techniques have been proposed to reduce SAR. These include lowering the flip angle (from 180°) in a refocusing train (TSE) [1] and stretching the RF pulses and modulating the refocusing train [2–4]. Each of these techniques has drawbacks. Modulation of refocusing RF pulses in a train as described in [2] and [4] is useful only for TSE sequences with long echo trains and cannot be used for steady-state sequences (SSFP) and other SAR sensitive sequences. Lowering the flip angle results in relatively lower signal-to-noise ratio while stretching RF pulses can increase echo spacing resulting in blurring and unwanted increase in T_2 effects. In addition, decreased bandwidths result in increased sensitivity to off-resonance spins.

A method to reduce SAR for any given RF pulse, termed ‘‘variable rate selective excitation’’ or VERSE, was originally proposed by Conolly et al. [5]. The VERSE technique is based on the idea of RF scaling [5], which will be described in more detail in the next section. When RF scaling is employed, the RF and gradient waveforms of a given RF pulse are modified, such that the excited slice profile is unchanged. The original RF pulse is referred to as the

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"constant-rate RF pulse", since it is employed with a constant gradient amplitude. RF scaling can be used to reduce SAR and peak b_1 amplitude [6–8] while preserving the original RF pulse duration. This is very important for high field sequences where SAR reduction is critical. Another application is the reduction of RF pulse duration, at the expense of increased SAR [9,10]. RF pulse width reduction is beneficial for sequences of very short TR such as Steady-State Free Precession [10]. Finally, RF scaling can be used to improve the slice profile of adiabatic inversion RF pulses [11].

Matson [9] proposed an algorithm to implement VERSE. In this algorithm, the RF scaling factor is recalculated at each discrete point in the RF and gradient waveforms. The Matson algorithm starts by applying an arbitrary RF scaling factor to the first point, and then proceeds to all the other points of the waveforms, such that the maximum gradient amplitude, slew rate and peak b_1 are not exceeded. The Matson algorithm was employed to reduce SAR while preserving the RF pulse duration [7,8] and to minimize RF pulse duration while increasing SAR [9,10].

In this paper, we propose an alternative approach to calculate optimal RF scaling, by using an analytic function. The parameters of this function are optimized, based on the application, i.e., minimum SAR or minimum RF pulse duration. We show that this approach has advantages over Matson's method, while providing identical or similar SAR and RF pulse width reduction. As with any VERSE approach, the slew rate will vary along the gradient waveform.

Maintaining the final pulse width at the initial pulse-width duration allows for re-use of the original canned protocols without the need to optimize the new sequence for a desired contrast.

2. Theory and methods

2.1. Review of VERSE

To better understand the solution presented below, it is instructive to review the VERSE technique [5].

Suppose we are given an RF pulse waveform, $b_1(t)$ Gauss, and a constant slice-select gradient waveform, $g(t)$ Gauss/cm, with N points. The excited slice width SW in cm is $\mathrm{SW} = \frac{\mathrm{BW}}{\gamma \cdot g(t)}$, where BW is the bandwidth of the RF pulse in kHz and $\gamma = 4.257$ kHz/Gauss for protons. A known method to reduce peak b_1 while maintaining SW is RF pulse stretching, where g is multiplied by a factor $G < 1$, and BW is reduced by the same factor. The reduction of BW is achieved by stretching b_1 in time by $1/G > 1$ and multiplying b_1 by G, thereby reducing peak b_1. Similarly, the RF pulse duration can be decreased by multiplying $g(t)$ and BW by a factor $G > 1$, thereby increasing peak b_1 by G. This is called RF pulse shrinking. The multiplication of b_1 and g by an arbitrary factor G while preserving SW is called RF scaling. In general, G may vary in time during the RF pulse, such that RF scaling is defined by the function $G(t)$. In VERSE, RF stretching ($G < 1$) is applied during the central lobe of the RF pulse where b_1 is high, and RF shrinking to the beginning and end of the RF pulse where b_1 is low. If done to maintain the original RF pulse duration the shrinking must compensate the stretching, such that the overall duration remains the same. In order to preserve the original RF pulse duration T, $G(t)$ must fulfill $T = \int_0^T G(t) \cdot \mathrm{d}t$ or equivalently

$$\mathrm{sum}(G(t)) = N \tag{1}$$

where sum( ) is the sum of all the points of $G(t)$, and N is the number of points. Eq. (1) implies that during RF scaling, the constant gradient $g(t)$ is multiplied by G, and the time increment $\mathrm{d}t$ between points by $1/G$ (as explained above) such that $G(t) \cdot \mathrm{d}t$ is preserved. The constraint in (1), as well as the maximum gradient amplitude and slew rate, is imposed during the calculation of G.

During stretching the time $\mathrm{d}t$ between points increases while the gradient and RF amplitudes decrease. During shrinking $\mathrm{d}t$ decreases while the gradient and RF amplitudes increase. The original g and b_1 waveforms are sampled at equidistant time intervals. Therefore, the scaled time-dependent waveforms are sampled at a varying rate. Since all waveforms must be sampled at equidistant intervals, one must calculate via interpolation new equidistant points on the scaled RF and gradient waveforms. After the calculation of $G(t)$, the RF and gradient waveforms are computed in two steps: (1) the original b_1 and g are interpolated. The time interval between adjacent interpolated points (on the original time axis) is proportional to $G(t)$. (2) The interpolated waveforms are multiplied by the scaling function $G(t)$.

In some types of RF pulses, e.g., adiabatic RF pulses, the waveforms are given by b_1, g, and a frequency waveform v_1. In this case, the VERSE algorithm is applied to all waveforms in the same way, i.e., v_1 is also interpolated and multiplied by $G(t)$.

When a conventional RF pulse excites an off-center slice at Δz cm from iso-center, a frequency offset $f = \gamma \cdot g \cdot \Delta z$ is applied, where g is the constant amplitude slice-selection gradient. Similarly, during a VERSE RF pulse, a time-dependent frequency waveform

$$\Omega_1 = \gamma \cdot G(t) \cdot g \cdot \Delta z \tag{2}$$

is applied during excitation. For off-center slices of an adiabatic VERSE RF pulse, the frequency waveform f is the sum of v_1 and Ω_1, i.e., $f = v_1 + \Omega_1$. An equivalent RF pulse can be generated by using a phase waveform Θ instead of the frequency waveform f, where

$$\Theta(t) = 2\pi \cdot \int_0^t f(t') \cdot \mathrm{d}t' = 2\pi \cdot \int_0^t [v_1(t') + \Omega_1(t')] \cdot \mathrm{d}t'$$

2.2. Calculation of G(t)

As explained before, $G(t) > 1$ at the beginning and end of the RF pulse and $G(t) < 1$ at the center where b_1 is high. To calculate G analytically, we select a smooth analytic function that fulfills these requirements, and then optimize its parameters based on gradient constraints and the constraint in (1). We select the function

$$G = (A - B) \cdot F_n + B \tag{3}$$

where F_n varies from 0 to 1. G is maximum at the beginning and end where $F_n = 0$ and $G = B$. G is minimum at the center where $F_n = 1$ and $G = A$. Hence $B \geqslant 1$ and $A \leqslant 1$.

F_n is based on the Fermi function

$$F_i(x) = \frac{1}{1 + \exp[(|x| - 1)/t_c]} \tag{4}$$

where $-1 \leqslant x \leqslant 1$ and t_c is a parameter to be adjusted. The minimum value for F_i is 0.5 at $|x| = 1$. The maximum is at $x = 0$:

$$F_i(0) = \frac{1}{1 + \exp(-1/t_c)} = a \tag{5}$$

F_n is obtained by stretching F_i to the [0 1] range, i.e.,

$$F_n = \frac{F_i - \min(F_i)}{\max(F_i) - \min(F_i)} = \frac{F_i - 0.5}{a - 0.5} \tag{6}$$

From the definitions in Eqs. (3)–(6), the scaling function G has three parameters that need to be optimized: A, B, and t_c. B is calculated from A and t_c by imposing the constraint (1). Fig. 1 shows G with $N = 400$ points for $A = 0.5$ and $t_c = 0.15$, 0.26, and 0.5. The calculated B is 2.9, 2.1, and 1.7, respectively.

The more A is reduced (at the expense of increased B), the more peak b_1 and SAR are reduced. However, as A is reduced, the slice profile of off-resonance spins becomes more distorted. The increased RF pulse stretching at a lower A reduces RF bandwidth and thereby increases frequency shifts of off-resonance spins. Therefore, if minimum SAR is desirable, A must be optimized such that SAR is reduced while maintaining an acceptable off-resonance slice profile. In the optimization described below, we assume a given $A < 1$. The parameters B and t_c are calculated using the maximum allowed gradient and slew rate, and the constraint of Eq. (1). The VERSE waveforms are calculated from the original b_1 and g by interpolation and multiplication by G as explained above, and the slice profile for fat spins (off-resonance of 3.5 ppm) is calculated. This process is repeated for a range of A values. The lowest A with an acceptable slice profile for fat spins is selected.

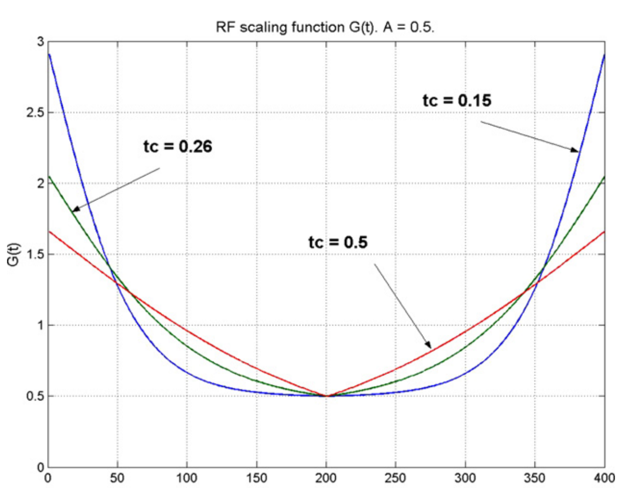

Fig. 1. RF scaling function, $G(t)$ (Eq. (3)), plotted for different values of t_c and a fixed $A = 0.5$. The B values (2.9, 2.1, and 1.7) are calculated by imposing the constraint (1).

2.3. Calculation of B and t_c using a given A

Given a waveform G (Eq. (3)) with N points for $|x| \leqslant 1$, the maximum increment of G between adjacent points in the waveform, defined as ΔG, is at $|x| = 1$. We define this value as *slew*, which is a unit-less number and is proportional to the maximum gradient slew rate. It can be shown that

$$slew = \Delta G(|x| = 1) = \frac{B - A}{(2a - 1)} \cdot \frac{1}{N.t_c} \tag{7}$$

where a is defined in Eq. (5). The maximum allowed slew rate in Gauss/cm/msec is equal to $slew \cdot g/\mathrm{d}t$, where g is the constant gradient amplitude in Gauss/cm and $\mathrm{d}t$ is the time between points in msec. From (4) and the definition of F_n in (5) and (6), the sum of the points in G (must be equal to N) is given by

$$\sum_i G_i = (A - B) \cdot \sum F_n + B \cdot N$$
$$= (A - B) \cdot \frac{N \cdot I - N}{2a - 1} + B \cdot N = N \tag{8}$$

where

$$I = \int_{-1}^{1} \frac{\mathrm{d}x}{1 + \exp[(|x| - 1)/t_c]} = 2 - 2t_c \cdot \ln(2a)$$

Using (7), we express (8) in terms of t_c and slew. The result is

$$f = 2 \cdot t_c^2 \cdot \ln(2a) - 2 \cdot t_c \cdot (1 - a) - K = 0 \tag{9a}$$

$$\frac{\mathrm{d}f}{\mathrm{d}t_c} = 4 \cdot t_c \cdot \ln(2a) + \left(4 + \frac{2 \cdot a}{t_c}\right) \cdot (a - 1) \tag{9b}$$

where $K = (1 - A)/(N \cdot \text{slew})$.

Eq. (9a) is solved by the Newton–Raphson method [12]. Starting with some initial value for t_c, we calculate f and vary t_c using (9b), until f is zeroed. Alternatively, (9a) can be solved with the bisection method ([12] pp. 246–247) if the derivative in (9b) cannot be derived analytically. With both methods, t_c converges quickly in a few iterations. After convergence, B is calculated by substituting for t_c in (7) or (8). It can be shown that a solution to (9a) is guaranteed if and only if slew $\geqslant 4 \cdot (1 - A)/N$. For all practical waveforms, this slew is very low and can be neglected.

Eqs. (3) to (9) assume a symmetric b_1 and G. The algorithm can be extended to asymmetric waveforms by shifting F_i in (4) along x. To shift F_i by d, we replace x in Eq. (4) with $(x - d)$. This algorithm is described in Appendix A.

2.4. $B > B_{max}$

B is restricted by the maximum allowed gradient amplitude $g_{\max}$ Gauss/cm, such that $B_{\max} = g_{\max}/g$, where g is the original constant gradient and $B_{\max}$ is the maximum allowed RF scaling. If the calculated B exceeds $B_{\max}$, then G must have N_2 points with a constant $B = B_{\max}$, and the number of points of F_n in Eq. (6) decreases to $N_1 = N - N_2$, as shown in Fig. 2. In this case we repeat the optimization with the parameters N_1 and t_c, and a fixed $B = B_{\max}$. Eqs. (7) and (8) are rewritten with N replaced by N_1 and B by $B_{\max}$:

$$\mathrm{sum}(G) = (A - B_{\max}) \cdot \frac{I \cdot N_1 - N_1}{2a - 1} + B_{\max} \cdot N = N \tag{10a}$$

$$slew = \frac{B_{\max} - A}{2a - 1} \cdot \frac{1}{N_1 \cdot t_c} \tag{10b}$$

To solve for t_c we substitute (10b) in (10a):

$$f = 1 - 2 \cdot t_c \cdot \ln(2a) - K_2 \cdot t_c \cdot (2a - 1)^2 = 0 \tag{11a}$$

$$\frac{\mathrm{d}f}{\mathrm{d}t_c} = -2 \cdot \ln(2a) + \frac{2 \cdot (1 - a)}{t_c} - K_2 \cdot (2a - 1)^2 + K_2 \cdot 4 \cdot \frac{(2a - 1) \cdot a \cdot (1 - a)}{t_c} \tag{11b}$$

where $K_2 = (B_{\max} - 1)/(B_{\max} - A)^2 \cdot N \cdot slew$.

We solve (11a) with the Newton–Raphson method using the derivative in (11b) or with the bisection method. This solution yields the optimal t_c. N_1 is calculated from (10b).

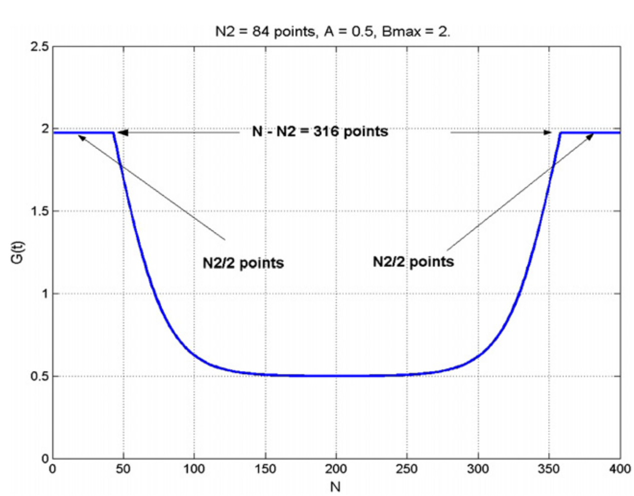

Fig. 2. Example of the scaling function, $G(t)$, when $B > B_{\max}$. There are $N_2 = 84$ points on the flat portion where $B = B_{\max} = 2$, $A = 0.5$ and the total number of points is $N = 400$.

As before, a solution is guaranteed if slew $\geqslant 4 \cdot (1 - A)/N_1$. Since the gradient waveform in Fig. 2 is symmetric, $N_2 = N - N_1$ must be an even integer. Therefore, the calculated N_1 is rounded, so that N_2 becomes an even integer. To ensure that (10a) is accurately fulfilled after the rounding, the value of B is slightly changed to:

$$B = \frac{N - A \cdot \mathrm{sum}(F_n)}{N - \mathrm{sum}(F_n)} \cong B_{\max} \tag{12}$$

F_n (with N_1 points) is defined in (6).

2.5. Optimization of $G(t)$ for maximum SAR reduction

In this section, our purpose is to minimize SAR while preserving RF pulse width, for given constant-rate waveforms b_1 and g and slice width SW, without exceeding the maximum slew rate and gradient amplitude. The SAR of any RF waveform with N points is proportional to

$$\mathrm{SAR} \propto \sum_{i=1}^{N} |b_{1i}|^2 \tag{13a}$$

where b_{1i} is the amplitude at point i. We define the SAR reduction of a VERSE waveform compared with the constant-rate waveform as:

$$\mathrm{SAR\ reduction}\ \% = \frac{\mathrm{SAR}_{\text{constant rate}} - \mathrm{SAR}_{\text{VERSE}}}{\mathrm{SAR}_{\text{constant rate}}} \cdot 100 = \frac{\sum |b_{1i}|^2 - \sum |B_{1i}|^2}{\sum |b_{1i}|^2} \cdot 100 \tag{13b}$$

where b_1 and B_1 are the constant-rate and VERSE waveforms, respectively.

From (7) and (10b), t_c is proportional to $1/slew$. From Fig. 1, $G(t)$ with smaller t_c will have a higher amplitude at the beginning and end of the RF pulse. Therefore, when we use a high slew rate, t_c is small and b_1 is higher at the beginning and at the end. This argument shows that for b_1 with significant amplitude at the beginning and end, the optimal slew rate for minimum SAR will be lower than for b_1 that is concentrated around the center. This effect is demonstrated in Section 3 below. Therefore, for an arbitrary b_1 the SAR must be calculated at all slew rates, and the optimal slew rate with minimum SAR is selected.

The optimization proceeds as follows: (1) For a given A, calculate waveforms and SAR for slew rates from the minimum $(4 \cdot (1 - A)/N)$ to the maximum. Since SAR vs. slew is a smooth function, calculate for about 10 points and interpolate. (2) Select the slew rate with minimum SAR. (3) Repeat this process with four or five A values, $0.4 \leqslant A \leqslant 0.8$. In each case, examine the off-resonance slice profile. Select the minimum A with acceptable off-resonance slice profile, since lower A yields lower SAR. For most RF pulses we use $A = 0.6$ for an off-resonance frequency of 450 Hz (fat at 3 T). If the fat signal is suppressed the A value can be reduced and the SAR reduction increases significantly, as demonstrated by the design of a 180° RF pulse in Section 3.

2.6. Optimization of $G(t)$ for minimum RF pulse duration

In this section we shall design $G(t)$ so as to minimize the RF pulse duration at the expense of increased SAR without exceeding the maximum b_1, slew rate and gradient amplitude.

Suppose we have designed a constant-rate RF waveform with duration T, peak b_1 value B_{1P} and constant gradient g. We would like to reduce the RF pulse duration to $\Im$ without exceeding the maximum allowed peak b_1 value B_{1A}. We define $F = T/\Im$. The design process is done in three steps. (1) Use the algorithm described above (Eqs. (3) to (12)) to design $G(t)$ with $A = B_{1A}/(B_{1P} \cdot F)$, maximum gradient amplitude g_{max}/F and maximum slew rate $slew/F^2$, where g_{max} is the maximum allowed gradient and *slew* is the maximum allowed slew rate. (2) Multiply $G(t)$ and the b_1 waveform from step (1) by F, and decrease the time dt between points by the same factor F. (3) Decrease $\Im$ (increase F) until the peak b_1 reaches the maximum allowed b_1 value B_{1A}. An example of a minimum-duration RF pulse design is given in Section 3.

The rationale for this algorithm is as follows: The A value is the scaling factor at the peak b_1. Our VERSE algorithm must modify the peak b_1, B_{1P}, of the original RF pulse to B_{1A}/F such that it becomes B_{1A} after scaling by F. Therefore, we design a VERSE RF pulse with $A = B_{1A}/(B_{1P} \cdot F)$ and a duration T. The whole RF pulse is then scaled by F, by multiplying the waveforms by F and decreasing dt (the time between points) by F. During scaling the maximum gradient amplitude increases by F, so we limit the gradient amplitude during the RF design step to g_{max}/F. The slew rate increases by F^2 after scaling, because the gradient increases by F and the time decreases by F. Therefore, we limit the maximum slew rate during the RF design step to $slew/F^2$.

The slice profile of spins at off-resonance is usually not a concern for the minimum-duration RF pulse, since in all practical designs $B_{1A}/B_{1P} \geqslant 0.7$. Therefore, we did not calculate the off-resonance slice profile of the minimum-duration RF pulse example in the next section.

3. Results

In this section, we shall demonstrate the minimum SAR VERSE optimization with a 180° RF pulse, an adiabatic inversion RF pulse and the minimum duration VERSE implementation with another 180° linear phase RF pulse.

3.1. 180° RF pulse

We optimize a linear phase 180° RF pulse designed with the SLR algorithm [13]. The RF pulse parameters are: bandwidth = 1.5 kHz, pulse width = 3.2 ms, in/out slice ripple = 1% and slice width SW = 3 mm. The maximum gradient is 5.0 Gauss/cm and maximum available slew rate is 150 T/m/s.

We optimize the waveforms for minimum SAR and an acceptable slice profile at an off-resonance frequency of 450 Hz (fat frequency at 3.0 T) with $A = 0.6$. The RF waveform and the SAR (Eq. (13a)) are calculated using Eqs. (3–11) for different slew rates, from the minimum (3.9 T/m/s) to the maximum (150 T/m/s). The result is shown in Fig. 3, where the minimum SAR (at maximum slew rate) is normalized to unity. The SAR is minimum at the maximum slew rate. This is expected because the energy of the constant-rate b_1 waveform (Fig. 4a) is concentrated at the RF pulse center. The optimized VERSE waveforms b_1 and G are shown in Fig. 4a and b, The SAR reduction as defined by Eq. (13b) is 39% and peak b_1 decreases from 0.25 to 0.15 Gauss. Fig. 5 shows M_z after the RF pulse for on-resonance spins and for spins at off-resonance of $\Delta f = 450$ Hz (fat at 3 T). The distortion of the off-resonance slice is acceptable. Notice the increased frequency

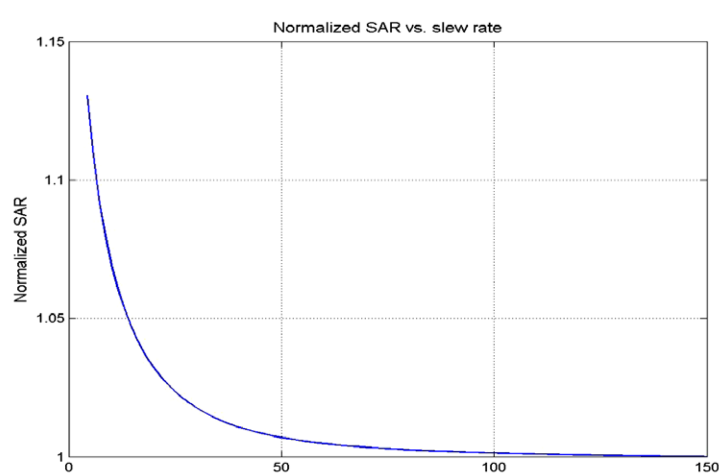

Fig. 3. Normalized SAR as a function of slew rate for the 180° linear phase RF pulse. The minimum SAR is obtained at the maximum slew rate of 150 T/m/s.

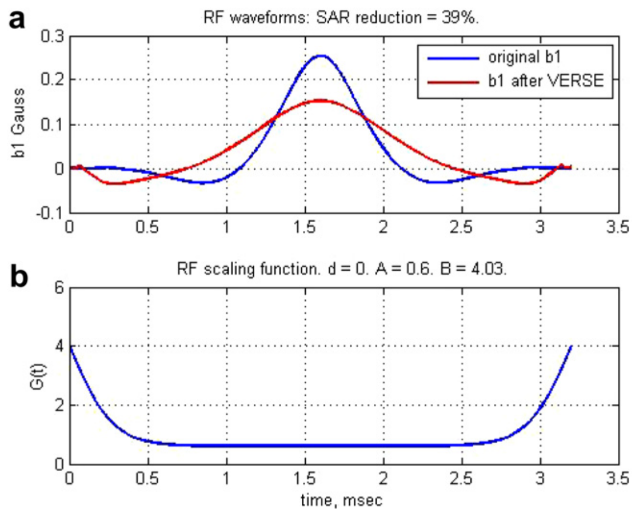

Fig. 4. (a) Optimized RF waveform and scaling function $G(t)$ for the 180° linear phase RF pulse, using a slew rate of 150 T/m/s, $A = 0.6$, and $B = 3.733$.

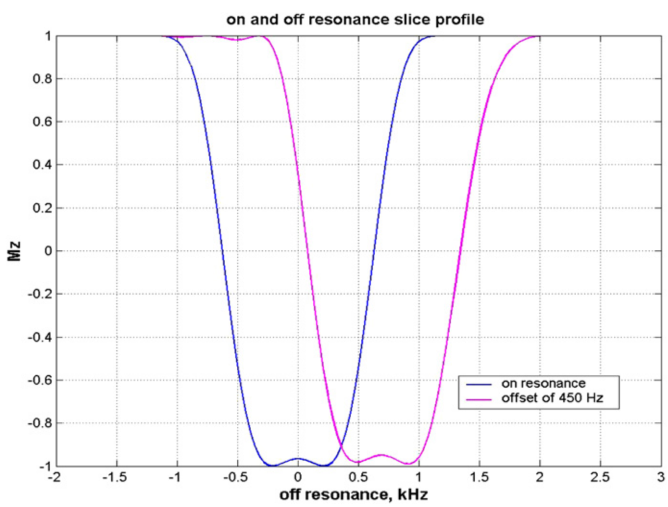

Fig. 5. Magnetization at resonance and off-resonance (450 Hz) after application of the pulses in Fig. 4. The off-resonance slice is shifted by $\Delta f/A = 750$ Hz.

shift of the off-resonance slice by about $\Delta f/A = 750$ Hz [5], due to the low gradient amplitude at the RF pulse center. Table 1 shows the SAR and peak b_1 reduction at $A = 0.4$, 0.5, and 0.6. The off-resonance column indicates the largest off-resonance where $M_z \leqslant -0.95$ at the shifted slice center. For a off-resonance frequency greater than the value shown, $M_z > -0.95$. The optimization is repeated for $A = 0.5$, 0.7, and 0.8. At $A = 0.5$ the off-resonance distortion at fat frequency is too high, so $A = 0.6$ is selected. If fat signal is suppressed, the selected A decreases because the expected off-resonance decreases. Consequently, the SAR is significantly reduced. We found that $A = 0.4$ is an acceptable value if fat signal suppression is employed.

Table 1
SAR and peak b_1 reduction for different A values for the RF pulse in Fig. 4

A	Off-resonance (Hz)	SAR reduction (%)	Peak b_1 (Gauss)
0.6	450	39	0.150
0.5	300	47	0.127
0.4	100	55	0.100

A, the minimum RF scaling is equal to the peak b_1 reduction. The off-resonance frequency is the largest frequency with $M_z \leqslant -0.95$ at the shifted slice center. The SAR reduction is defined in Eq. (13b) as the SAR without VERSE minus with VERSE divided by the SAR without VERSE.

3.2. Adiabatic inversion RF pulse

The algorithm is applied to a Silver–Hoult [14] adiabatic inversion RF pulse, which employs a b_1 waveform and a frequency waveform v_1. The b_1 and the frequency waveforms are:

$$b_1 = b_0 \cdot \mathrm{sech}(\beta t) \tag{14a}$$

$$v_1 = \mathrm{bw}/2 \cdot \tanh(\beta t) \tag{14b}$$

where bw is the bandwidth, and b_0 is the maximum b_1 amplitude. The time t is defined from $-T/2$ to $T/2$, where T is the RF pulse width. The RF pulse parameters are: bw = 1.6 kHz, $b_0 = 0.12$ Gauss, RF pulse width = 8.64 ms, and $\beta = 0.694$ kHz. b_1 and v_1 vs. t are shown in Fig. 6. The slice width SW is 4 mm, with maximum gradient and slew rate of 4.0 Gauss/cm and 150 T/m/s, respectively.

Our purpose is to minimize SAR with an acceptable profile at $\Delta f = 450$ Hz. Fig. 7 shows the SAR vs. slew rate for this RF pulse with $A = 0.6$. The minimum SAR is not at the maximum slew rate, because b_1 at the end and beginning of the RF pulse (Fig. 6) is not negligible (0.012 Gauss). The optimal slew rate (for minimum SAR) is 36 T/m/s. At this slew rate, the ratio between the SAR of the constant-rate to the VERSE'd b_1 is 1.35 and the peak b_1 decreases from 0.12 to 0.071 Gauss. Fig. 8 shows the VERSE b_1 and frequency v_1 waveforms at this optimal slew rate. Fig. 9a shows the M_z slice profile for the optimal slew rate at an offset of 0 and 450 Hz. The off-resonance slice is distorted due to the high amplitude (0.04 Gauss) at the beginning and end of the b_1 waveform in Fig. 8. To reduce these "wings" we increase t_c (reduce slew rate) even more, at the expense of increased SAR. We reduce slew rate to 2.6 T/m/s and obtain the b_1 and v_1 waveforms in Fig. 8, with acceptable off-resonance slice profile that is shown in Fig. 9b. The "wings" at the beginning and end of the RF pulse are now 0.019 Gauss, but the SAR increases by 10% (see Fig. 7), and the peak b_1 is 0.084 Gauss. The SAR of the low slew rate b_1 (Fig. 8) is 20% lower than the SAR of the original constant-rate b_1 waveform.

3.3. Minimum duration 180° linear phase RF pulse

We design a minimum-duration RF pulse, using the same parameters as Matson (reference [9] Fig. 6a). This is a linear phase 180° RF pulse with duration and bandwidth of 5.38 ms and 2.0 kHz, respectively. The constant slice-select gradient is 0.4 Gauss/cm and the maximum gradient

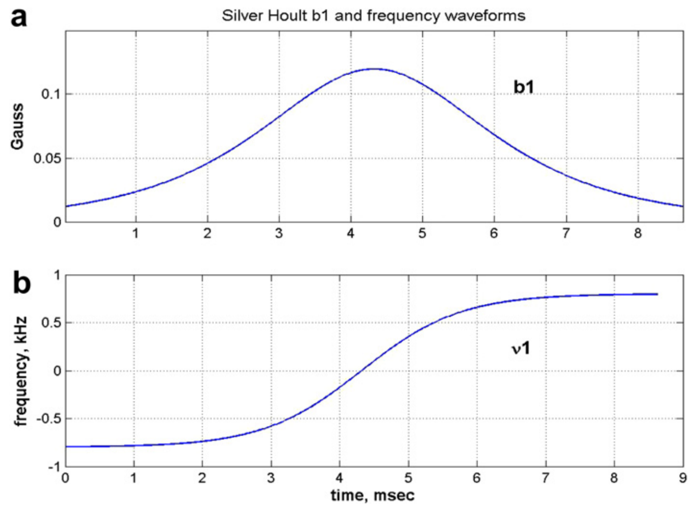

Fig. 6. Silver–Hoult adiabatic inversion RF pulse with a bandwidth of 1.6 kHz and duration of 8.64 ms. (a) RF profile (b_1) waveform and (b) frequency sweep ($\nu 1$) waveform.

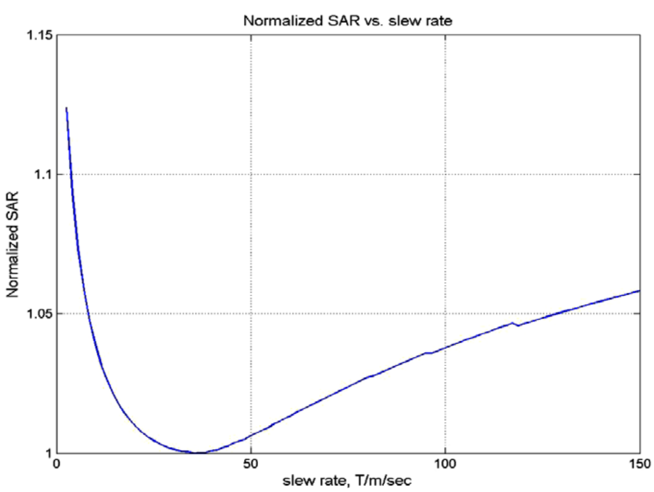

Fig. 7. Normalized SAR as a function of slew rate for the Silver–Hoult RF pulse of Fig. 6. The slew rate with minimum SAR is 36 T/m/s.

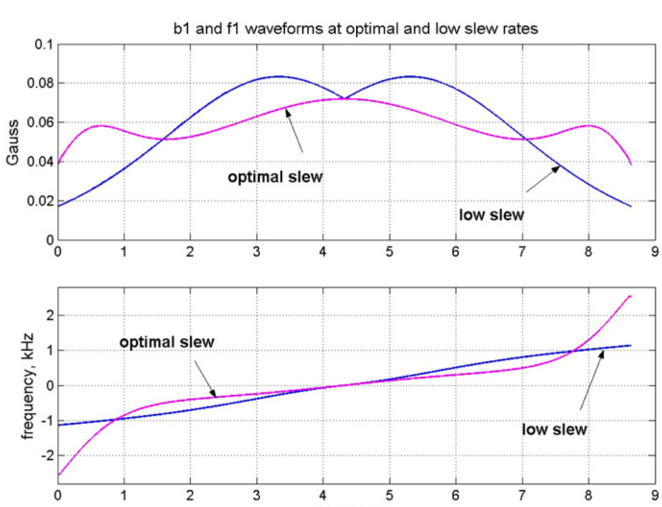

Fig. 8. b_1 and frequency sweep waveforms after VERSE with two different slew rates: the optimal slew rate of 36 T/m/s and the low slew rate of 2.6 T/m/s with improved off-resonance slice profile.

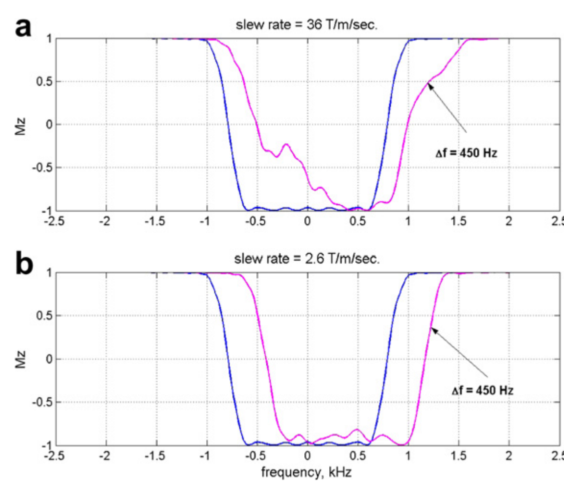

Fig. 9. Magnetization (M_z) after the application of Silver–Hoult RF pulse with a frequency offset of 0 and 450 Hz. (a) Slew rate of 36 T/m/s and (b) slew rate of 2.6 T/m/s.

amplitude and slew rate are 1.0 Gauss/cm and 40 T/m/s, respectively. B_{1P}, the peak b_1 of the constant-rate RF pulse is 0.45 Gauss. The maximum allowed b_1 is $B_{1A} = 0.32$ Gauss. The VERSE RF and gradient waveforms calculated with the Matson's algorithm are shown in Fig. 10(A) and (B) in reference [9]. The RF pulse duration decreased from $T = 5.38$ ms to $\Im = 2.5$ ms, so $F = 5.38/2.5 = 2.15$.

The RF pulse is designed in two steps. *Step 1*: design a $T = 5.38$ ms RF pulse with $A = B_{1A}/(B_{1P} \cdot F) = 0.32/(0.45 \times 2.15) = 0.33$, maximum gradient of $g_{max}/F = 0.46$ Gauss/cm and maximum slew rate slew/$F^2 = 8.6$ T/m/s. *Step 2*: multiply the gradient and b_1 waveforms by F and shrink the RF pulse by F, so its final duration is $5.38/F = 2.5$ ms. The resultant waveforms are shown in Fig. 10. As required, the peak b_1 is 0.32 Gauss, and the maximum gradient and slew rate are 0.995 Gauss/cm and 40 T/m/s, respectively. Reduction of $\Im$ below 2.5 ms, results in peak $b_1 > 0.32$ Gauss. Unlike Matson's, the

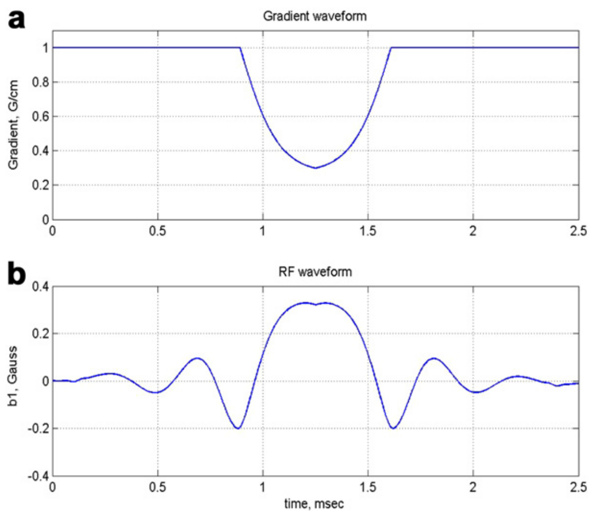

Fig. 10. RF and gradient waveforms of a linear phase 180° RF pulse with a bandwidth of 2 kHz and maximum b_1 of 0.32 Gauss. The RF pulse duration was reduced by RF scaling from 5.38 to 2.5 ms. The same reduction was obtained by Matson [9] in Fig. 10(A) and (B).

waveforms in Fig. 10 are smooth and can be downloaded to the MRI system without filtering.

3.4. Comparison with Matson's algorithm

RF scaling can be used for either reduction of SAR or reduction of RF pulse duration. An alternative algorithm to calculate the RF scaling function, $G(t)$, is Matson's algorithm [9]. We have compared the performance of our method to Matson's by (a) the achievable SAR reduction using a given A and RF pulse width T and (b) the minimum achievable RF pulse duration for a given maximum peak b_1.

3.4.1. SAR reduction

We used the RF pulse described by Busse et al. [8] with the following parameters: bandwidth 1.5 kHz, pulse width 1.84 ms, RF flip angle 130°, maximum slew rate of 150 T/m/s and gradient amplitude 40 T/m, slice width 8.8 mm and $A = 0.5$. The SAR reduction (as defined in (13b)) obtained with Matson's algorithm (Figs. 1 and 2 of reference [8]) is 22%. With our algorithm, we achieve 24% reduction. The difference is due to the lower gradient at $t = 0$ of 1.7 Gauss/cm in Fig. 2 of reference [8], compared to 3.7 Gauss/cm obtained with our method.

3.4.2. Reduction of RF pulse width

We have compared the minimum RF pulse width for a given maximum b_1 for three RF pulses: two are shown in Fig. 10 of reference [9], and the third is the RF pulse used by Hargreaves et al. [10], with TB = 10. Using the RF pulses from Matson's paper, we obtained the same pulse width and peak b_1 reduction. RF pulses shown in Hargreaves [10] are different since G is constrained to be zero at the beginning and end so the rise and fall of the gradient waveform are included in the waveform scaling. In Appendix B we extend our method to waveforms that include rise and fall. We apply this algorithm to the RF pulse in Fig. 1 of reference [10] and reduce the duration from 2.9 to 0.828 ms. This is close to the Matson algorithm in [10], where the pulse width was reduced from 2.9 to 0.8 ms.

3.5. Volunteer images

The low slew rate Silver–Hoult RF pulse of Fig. 8 was applied to a product 3T scanner (GE Healthcare, Waukesha, WI) sequence. Volunteers were scanned with different coils and protocols on a GE 3T Excite scanner equipped with a *TwinSpeed* gradient module (maximum amplitude and slew rate of 4 G/cm and 150 T/m/s). T1-Flair volunteer images of the head, pelvis, and knee are shown here.

Three sequences were compared: (a) standard IR FSE baseline sequence (SEQ 1), (b) standard IR FSE with slice select and refocusing RF pulses VERSE'd as in [7] (SEQ 2), and (c) IR modified with the current algorithm in conjunction with slice select and slice refocusing VERSE'd as in [7] (SEQ 3). Images obtained with SEQ 1 and SEQ 3 are shown here.

A T1-Flair image of the brain using an eight-channel head coil is shown in Fig. 11a. Scan parameters: TR = 2 s, TE = 17 ms, TI = 960 ms, ETL = 6, slice thickness/spacing = 4/1 mm, 27 slices. Images of the pelvis (TR = 2.6 s, TE = Min. Full, TI = 960 ms, ETL = 6, slice 6/1 mm, 10 slices) and knee (TR = 2.7 s, TE = min. Full, TI = 1 s, ETL = 6, slice 4/1 mm, 17 slices) using a TORSO phased array and a QUADKNEE coil, respectively, are shown in Fig. 11b and c, respectively. Fig. 12 shows the difference images for the three anatomical scans. Some differences between the VERSE and standard images are in regions where fat is displaced (off-resonance shift) along the slice select direction or due to slight misregistration between the two cases. Figs. 11 and 12 show that image quality is preserved when the conventional pulses are replaced with VERSE pulses.

Table 2 summarizes the reduction in SAR (defined by Eq. (13b) and measured by an external power monitor) and the increase in slice coverage between the three sequences. The first column shows SAR reduction for the same slice coverage, and the other columns show the number of slices per acquisition of both sequences with the same maximum SAR. The increase in slice coverage is not proportional to the decrease in SAR, because the VERSE sequence may go from being SAR limited to being limited by other constraints such as sequence time, gradient heating, etc.

4. Discussion and conclusion

In this paper, we have used a smooth analytic RF scaling function $G(t)$. The parameters of $G(t)$ (t_c, A and B) are optimized such that the maximum gradient amplitude and slew rate are not exceeded. The proposed algorithm is general and can be applied to any constant-rate RF pulse,

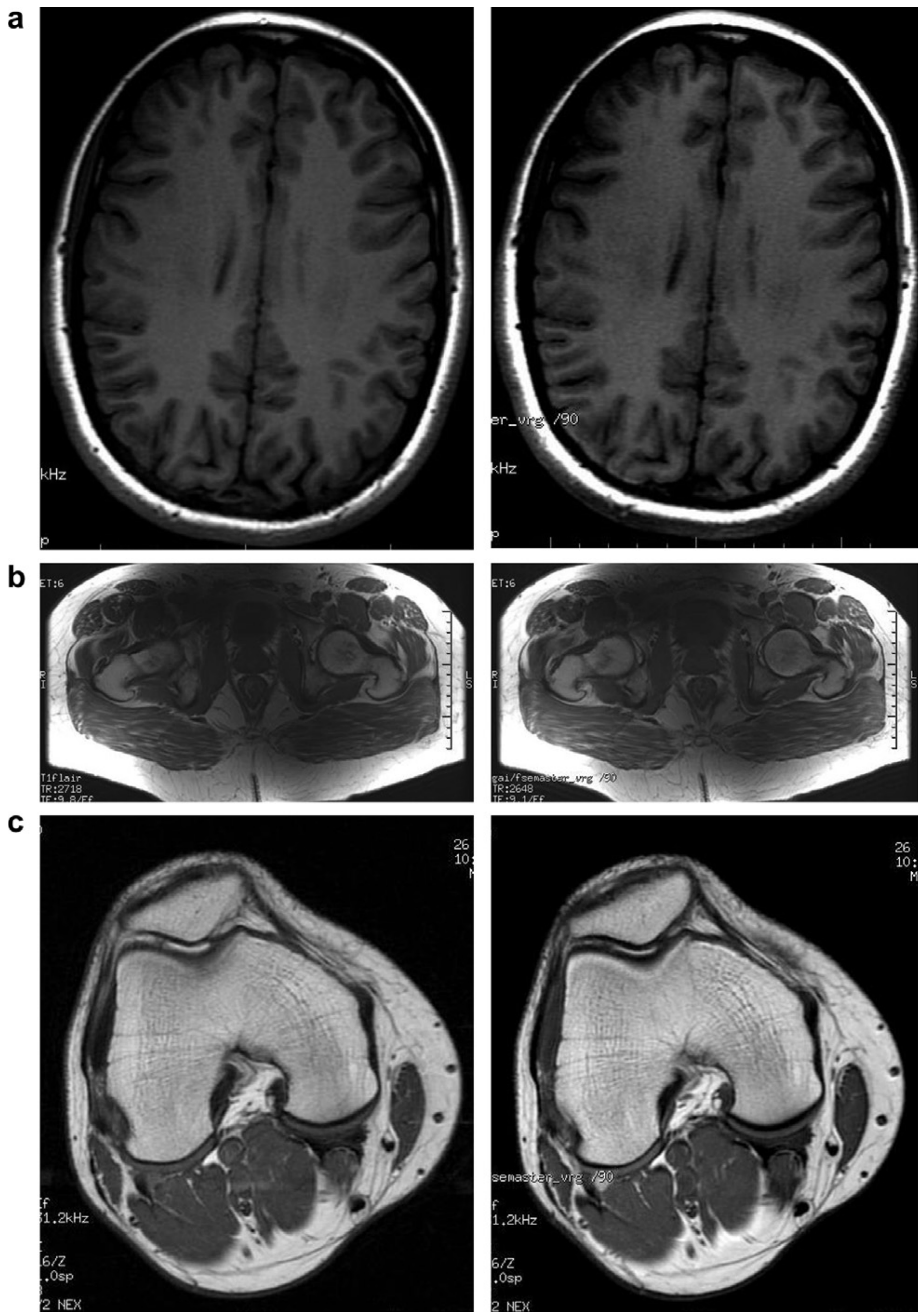

Fig. 11. T1-Flair images in the (a) head with SEQ 1 (left) and with SEQ 3 (right), (b) images obtained in the abdomen with SEQ 1 (left) and with SEQ 3 (right), and (c) images obtained in the knee with SEQ 1 (left) and with SEQ 3 (right).

including asymmetric RF waveforms (Appendix A) and gradient waveforms that include rise and fall (Appendix B). We have shown that we can use this method for either SAR reduction or for minimization of RF pulse length.

Matson [9] proposed an iterative algorithm to calculate $G(t)$, taking into account maximum gradient amplitude and slew rate. The Matson algorithm optimizes the waveforms on a point-by-point basis. Therefore, at each point the maximum possible scaling function (which is constrained by the maximum b_1, slew rate and gradient amplitude) is used. Hence, the calculation is iterative [9] and the waveforms may have sharp edges that may need filtering before downloading to the system. Our method is based on optimization of a global analytic function. As a result, the calculation is simpler and the waveforms are smooth and slowly-varying.

We have shown that the performance of both methods is similar, and the differences between them depend on the application. For the constant-duration RF pulses, our method seems to be superior because it automatically preserves the duration, whereas Matson's method adjusts the initial scaling function (G at $t = 0$) by trial and error until

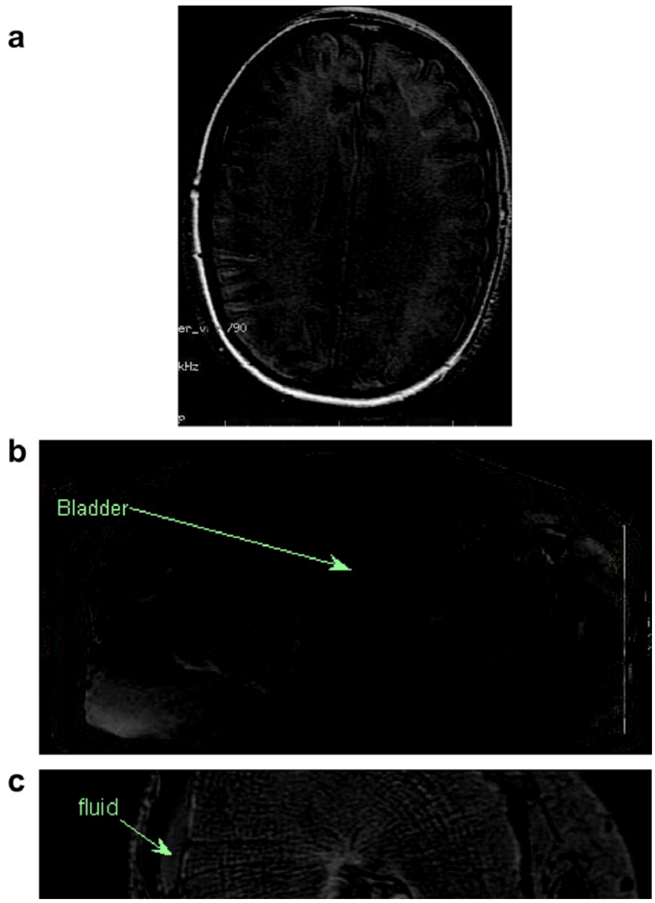

Fig. 12. Difference images for (a) head, (b) abdomen, and (c) knee $T1$ Flair showing negligible differences in the regions where fluid is present.

Table 2
Values reflect the reduction in SAR achieved when slice coverage is kept constant (under "SAR reduction")

	SAR reduction (%)			Maximum slices in TR		
	SEQ. 1	SEQ. 2	SEQ. 3	SEQ. 1	SEQ. 2	SEQ. 3
Head	0[a]	23	46	18	21	23
Pelvis	0	17	50	15	18	21
Knee	0	36	50	17	20	22

Alternately, the number of slices per acquisition (TR) increases with application of VERSE for the three different anatomical scans.
[a] Base sequence for comparison.

the desired duration is obtained. For the minimum-duration RF pulses, Matson's method is slightly superior because the scaling function is adjusted on a point-by-point basis, whereas our method uses an optimized global function. Despite the small differences, we think that in practice the performance of both methods can be considered as equivalent.

RF scaling reduces SAR and peak b_1 while increasing slice profile distortion for off-resonance spins, so that larger SAR reduction increases slice profile distortion for a given off-resonance frequency [6]. Therefore, the maximum SAR reduction factor that achieves an "acceptable" slice profile at the maximum expected off-resonance frequency (usually 3.5 ppm for fat) is selected. Since an "acceptable" slice profile is not an objective definition, the compromise between SAR reduction and off-resonance distortion depends on the judgment of the RF pulse designer. This compromise depends also on the application. For example, if fat signal suppression is applied the maximum off-resonance frequency is much lower than 3.5 ppm, so that SAR reduction may increase significantly as indicated by Table 1. Therefore, it is recommended that the RF scaling calculation be incorporated within the MRI machine software and will be calculated on the fly based on the current prescription, i.e., fat signal suppression and the expected field inhomogeneity within the sensitive region of the receiver coil.

Figs. 3 and 7 show that slew rate has relatively small effect on SAR reduction. In both cases, the SAR varies by about 10% over the entire range of allowed slew rates while the improvement in Fig. 7 between maximum slew rate (150 T/m/s) and the optimal slew rate (36 T/m/s) is 5%. Therefore, we usually assume that the optimal slew rate is the maximum. In case it is desired to find the optimal slew rate where SAR is minimal, we calculate SAR at only 5–10 different slew rates and interpolate. This is a fast calculation, because the scaling function G is analytic.

The choice of the analytic RF scaling function G in Eq. (3) is not unique. Some other smooth analytic function (such as a polynomial) could be used instead. The only requirement is that G must be high where the constant-rate b_1 waveform is low and low where the constant-rate b_1 is high.

The use of RF scaling based on a Fermi function (Eqs. (3) and (4)) allows SAR and peak b_1 reduction of an RF waveform with a single peak. In case of a constant-rate b_1 waveform with more than one peak, a separate Fermi function with optimized parameters can be used for each peak such that the overall SNR is minimized.

The volunteer images in Figs. 11 and 12 and results from [7] and [8], demonstrate that image quality from sequences with VERSE RF pulses is equivalent to image quality with conventional RF pulses. Therefore, VERSE RF pulses can replace conventional RF pulses and improve performance by decreasing SAR, increasing slice coverage and lowering TR.

Appendix A

We extend the derivation of Eqs. (3–9) to an asymmetrical RF pulse with N points. The RF scaling G is calculated by replacing x in Eq. (4) with $x - d$.

$$F_i = \frac{1}{1 + \exp[(|x - d| - 1)/t_c]} \tag{A1}$$

where $-1 \leqslant x \leqslant 1$ and $-1 \leqslant d \leqslant 1$. For $d > 0$ F_i is shifted to the right, and for $d < 0$ F_i is shifted to the left. We stretch F_i to the range [0 1]. The new function, F_n, is

$$F_n = \frac{F_i - \min(F_i)}{\max(F_i) - \min(F_i)} = \frac{F_i - b}{a - b} \tag{A2}$$

where $a = 1/[1 + \exp(-1/t_c)]$ is the maximum of F_i and $b = 1/[1 + \exp(|d|/t_c)]$ is the minimum of F_i. G is given by

$$G = (A - B) \cdot F_n + B \tag{A3}$$

From Eq. (1) the sum of G is equal to N:

$$\mathrm{sum}(G) = (A - B) \cdot \mathrm{sum}(F_n) + B \cdot N = N \tag{A4}$$

From (A2)

$$\mathrm{sum}(F_n) = \frac{I - 2b}{2a - 2b} \cdot N \tag{A5}$$

where $I = \int_{-1}^{1} F_i \cdot \mathrm{d}x = 2 - t_c \cdot \ln\left[a^2 \cdot (2 + 2 \cdot \cosh(d/t_c)\right]$

From (A4) and (A5):

$$\mathrm{sum}(G) = \frac{I - 2b}{2a - 2b} \cdot (A - B) \cdot N + B \cdot N = N \tag{A6}$$

The maximum increment of G, ΔG, is at $x = x_0$, where $F_i = 0.5$.

$$slew = \Delta G|_{x=x_0} = \frac{B - A}{(2a - 2b) \cdot 2 \cdot t_c} \cdot \frac{2}{N} = \frac{B - A}{2a - 2b} \cdot \frac{1}{N \cdot t_c} \tag{A7}$$

where

$x_0 = 1 + d$ for $d < 0$

$x_0 = -1 + d$ for $d > 0$

From (A6) and (A7)

$$f = \frac{A - 1}{N \cdot \mathrm{slew}} + t_c \cdot (2a - I) = 0 \tag{A8}$$

t_c is found by solving (A8) numerically using the bisection method (reference [12] p. 246). The solution converges very quickly in a few iterations. F_n is calculated by substituting t_c in (A2), and B is found from (A4) as

$$B = \frac{N - A \cdot \mathrm{sum}(F_n)}{N - \mathrm{sum}(F_n)} \tag{A9}$$

Finally, G is calculated using (A3).

Example: VERSE an asymmetrical minimum phase RF pulse with the following parameters: duration = 2.5 ms, bandwidth = 4 kHz, RF flip angle = 60°, $N = 625$ points and $A = 0.6$. The maximum gradient amplitude is 4 Gauss/cm and maximum slew rate is 150 T/m/s. From the constant-rate RF waveform we find the shift $d = 0.72$, where b_1 is a maximum. The constant-rate and VERSE'd waveforms are shown in Fig. A1a and the RF scaling G in Fig. A1b. The SAR reduction (Eq. (13b)) is 35%.

Appendix B

Here we calculate the RF scaling G, which is constrained to zero at the beginning and at the end of the waveform as shown in Fig. B2. The duration of the constant-rate RF pulse is T with N points, so the time between points is $\Delta t = T/N$.

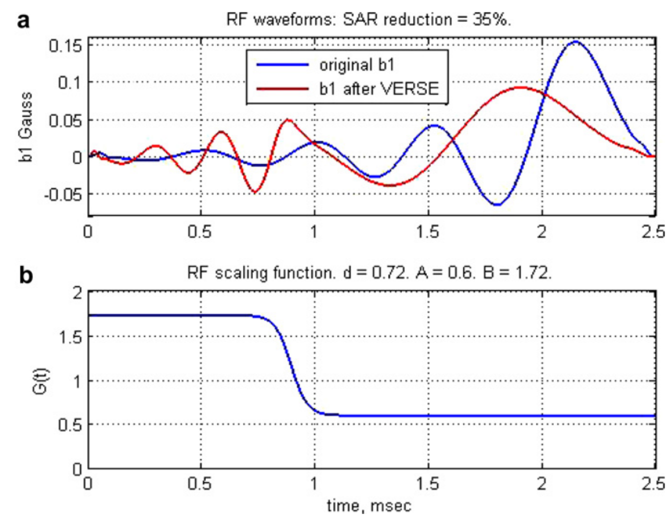

Fig. A1. VERSE of an asymmetrical minimum phase RF pulse with 2.5 ms duration, 4 kHz bandwidth, A = 0.6 and RF flip angle 60°. (a) The constant-rate and VERSE'd RF waveforms with SAR reduction of 35%. (b) The RF scaling function G. The shift parameter $d = 0.72$ is calculated such that the minimum of G is at the peak of the constant-rate RF waveform.

Suppose that the central part of G in Fig. B2 has N_1 points. The amplitude B can be calculated by solving Eq. (9a) for N_1 points such that the sum of the central part is N_1. The sum of all the points in G is given by:

$$\mathrm{sum}(G) = N = N_1 + B \cdot N_0 \tag{B1}$$

where N_0 is the number of points in the rise (fall) time. During rise time we use the maximum slew rate *slew*, so that

$$N_0 \cdot slew = B \tag{B2}$$

From (B1) and (B2):

$$f = N - N_1 - \frac{B^2}{slew} = 0 \tag{B3}$$

Since B can be calculated for any given N_1, we can solve (B3) numerically using the bisection method. This gives the shortest possible G for the given time between points Δt.

Next we reduce the RF pulse duration by a factor F as explained in the minimum RF pulse duration section. The calculation is repeated for a few F values, until the peak b_1 is approximately equal to the peak b_1 of the constant-rate RF pulse.

Example: We use the constant-rate RF pulse of Fig. 1 in Hargreaves et al. [10]: TB = 10, duration $T = 2.9$ ms, RF flip angle 60°, maximum slew rate 150 T/m/s, $\Delta t = 4$ µs and $N = 725$.

In the first step we obtain $N_1 = 448$ points, $N_0 = 92$ points and $B = 3.0$. In the second step we reduce the RF pulse duration by $F = 3.05$ to 0.828 ms. The constant-rate and VERSE'd b_1 waveforms are shown in

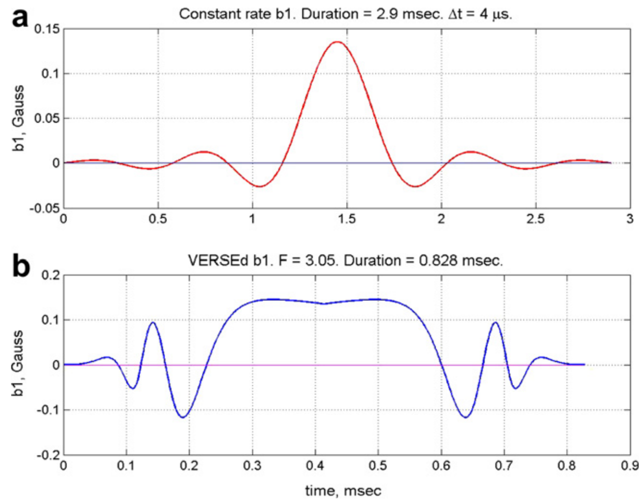

Fig. B1. Minimum-duration VERSE with gradient rise and fall. (a) Constant-rate b_1. $\mathrm{TB} = 10$, $T = 2.9$ ms, $\alpha = 60°$, and $\Delta t = 4$ μs. (b) Minimum-duration VERSE b_1. Duration = 0.828 ms. Note the different time scales in (a) and (b).

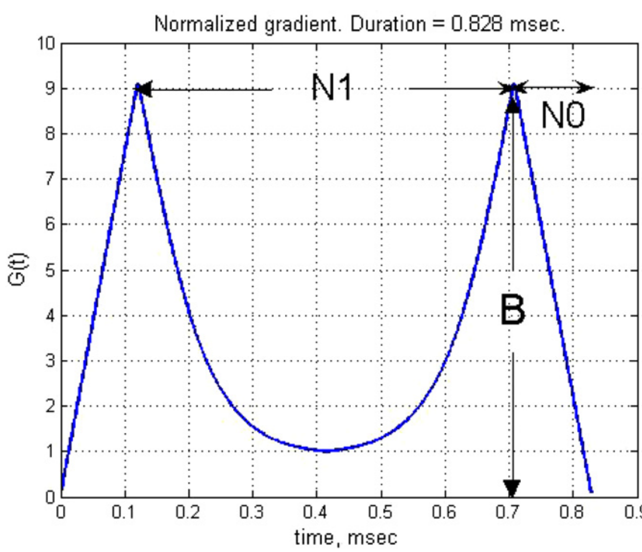

Fig. B2. RF scaling function for Fig. B1. There are $N_0 = 92$ points on the rise/fall and $N_1 = 448$ points on the central part. The time between points is $\Delta t = 1.31$ μs, and the maximum B is 9.15.

Fig. B1, and the scaling function G in Fig. B2. In the minimum-duration waveform $B = 3.0 \cdot F = 9.15$ and $\Delta t = 4$ μs/$F = 1.31$ μs.

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